Closing Wed: HW_9A,9B,9C (9.1/3/4,3.8)
Final Exam: Saturday, March 11th
1:30-4:20pm, Kane 130
(email me if you want a left-hand seat)

Entry Task:
Record the initial temperature of my hot water at the start of class and 10 minutes into class. We will try to predict the temperature at the end of class.

Note: Concerning differential equations, in this course you are expected to:

1. Solve separable equations.
2. Use initial conditions & constants.
3. Be able to set up the applied problems from homework.
4. Be able to work with a given applied differential equation or one where we step you through the set up.

Worried about the applied problems? Go through and look at the applied differential equation problems on the old finals (typically the last page).

9.4 Differential Equations Apps

Goal: Look at exponential growth/decay, Newton's law of cooling, and mixing problems.

***1. Law of Natural Growth/Decay*:**

Assumption: "*The rate of growth/decay is proportional to the function value*."

Example:

A population has 500 bacteria at $t = 0$. After 3 hours there are 8000 bacteria. Assume the population grows at a rate proportional to its size.
Find $B(t)$.

Example:

The *half-life* of cesium-137 is 30 years. Suppose we start with a 100-mg sample. The mass function $m(t)$ decays at a rate proportional to its size. Find $m(t)$.

2. Newton's Law of Cooling:

Assumption: "*The rate of temperature change is proportional to the difference between the temperature of the object and its surroundings*."

3. Air Resistance:

A skydiver steps out of a plane that is 4,000 meters high with and initial downward velocity of 0 m/s. The skydiver has a mass of 60 kg.

Let y(t) = "height at time *t*"

Let v(t) = y'(t) = "velocity at time *t*"

Let a(t)=v'(t)=''(t) = "accel. at time *t*"

Newton's 2nd Law says:

(mass)(acceleration) = Force

$$m\frac{d^2y}{dt^2} = \text{sum of forces on the object}$$

The force due to gravity has constant magnitude (and it is acting downward):

$F_g = -mg = -60 \cdot 9.8 = -588$ N

One model for air resistance

The force due to air resistance (*drag force*) is proportional to velocity and in the opposite direction of velocity. So

$$F_d = -k\ v \quad \text{Newtons}$$

Assume for this problem k = 12.

4. Mixing Problems:

Assume you have a vat of liquid that has a substance (a contaminant) entering at some rate and exiting at some rate, then

"*The rate of change of the contaminant is equal to the rate at which the contaminant is coming IN minus the rate at which it is going OUT.*"